



# SEA WAVES AND LONG PERIOD VERTICAL SEISMOMETERS

Lorenzo Papa<sup>(1)</sup>, Marina Pizzo<sup>(2)</sup>

<sup>(1)</sup>Istituto Idrografico Marina & University of Genoa, Dept. of Physics, Italy, E-mail: papa@fisica.unige.it

<sup>(2)</sup>University of Genoa, Dept. of Physics, Italy



## Introduction

In physical oceanography the measurement of sea waves is of great interest in the design of coastal structures, harbour planning and shoreline protection. Surf wave coast effects and deep water standing oscillations produce primary microseism waves with the same dominant period as ocean surface waves and secondary microseisms which have half this period. A long period vertical seismometer and a prototype of variable capacitance accelerometer have been used to compute in real time the sea wave spectrum in the Gulf of Genoa.

### Longuet-Higgins theory<sup>1</sup>

The pressure variations beneath a progressive gravity wave of Stoke's type are insufficient, in deep water, to generate microseisms of the observed magnitude. This is because the pressure variations on the sea-bed decrease exponentially with the depth. However, there are second order pressure variations which are not attenuated with the depth. The stationary wave profile at the depth  $h$ , in Lamb's notation, is given by:

$$\eta = a \cos(kx) \cos(\omega t) + O(a^3)$$

where  $a$ ,  $k$  and  $\omega$  are respectively amplitude, wave number and angular frequency which is equal to:  $\sqrt{gk \tanh(kh)}$ .

If we consider the water mass contained between the bottom, the surface and the two vertical planes, there isn't flow across the vertical planes. Therefore, if  $F$  is the vertical component of the total external force acting on the water mass, we have:

$$F = \sum m \frac{d^2 z}{dt^2} = \frac{1}{g} \frac{d^2 U}{dt^2} = \lambda(p - p_0 - \rho gh)$$

summing the equations of motion for each particle of mass  $m$  and cancelling internal forces ( $U$  potential energy,  $p_0$  pressure at the free surface,  $p$  mean pressure on the bottom,  $\rho$  water mean density,  $\lambda$  wave length).

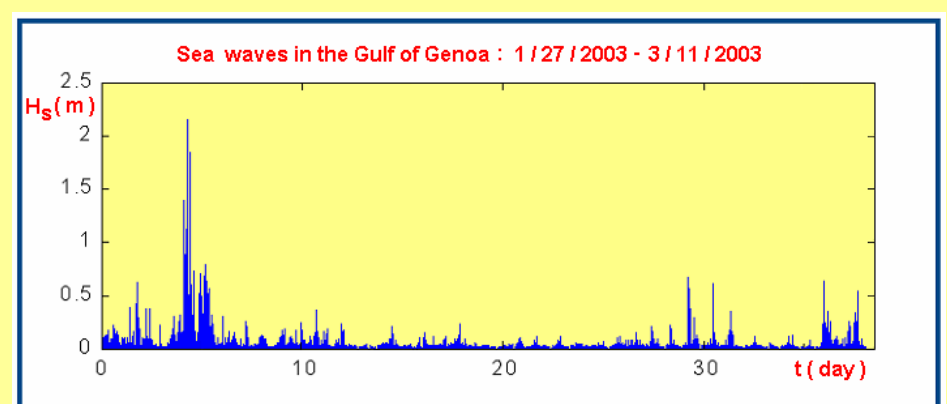
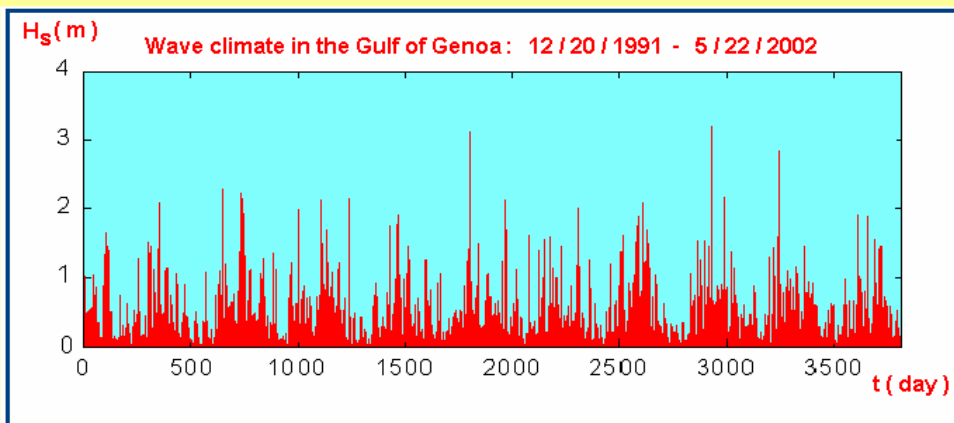
For the stationary waves (without compressibility) we have:

$$U = \int_0^\lambda \frac{1}{2} \rho g \eta^2 dx = \frac{1}{4} \lambda \rho g a^2 \cos^2 \omega t + O(a^3).$$

Hence: 
$$\frac{p - p_0}{\rho} = gh - \frac{1}{2} a^2 \omega^2 \cos^2 2\omega t + O(a^3).$$

The second term is of order of  $a^2$ , which explains why it is not revealed in the ordinary first order theory. By a slight extension of the present argument one can evaluate the mean pressure below the series of long-crested waves. Hence this type of pressure fluctuation occurs whenever wave-trains cross which are of the same frequency and travel in opposite directions.

Darbyshire, Okeke<sup>2</sup> and Korhonen<sup>3</sup> have supposed the fundamental and the second harmonic recorded by a seismometer are given by the wave impact on the bottom (shallow water effects).



## References

<sup>1</sup>M. S. Longuet-Higgins, F. Ursell, *Nature* **162**, 700 (1948)

<sup>2</sup>J. Darbyshire and E. O. Okeke, *Geophys. J. R. Astron. Soc.* **17**, 63-92 (1969)

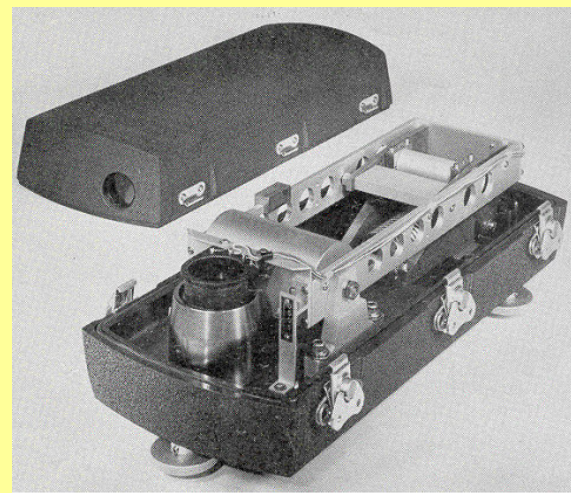
<sup>3</sup>H. Korhonen, *Spectral composition of microseisms recorded at seismograph station Oulu* UGGI Monogr. **31**, 33-52 (1970)

<sup>4</sup>L. Papa, 3<sup>rd</sup> AIOM Congress on marine and offshore engineering, Genoa (July 1992)

## Instruments

To measure sea waves in the Gulf of Genoa, we are using at the Department of Physics two instruments based on different physical principles:

- a long-period vertical seismometer; it employs a sensitive electromechanical transducer that converts long-period vertical motion into electrical output. This seismometer (Model SL-210, Teledyne Geotech) has been installed in 1989<sup>4</sup>;



- a prototype of variable capacitance accelerometer; it measures ground vertical accelerations caused by coastal sea waves. The variable capacitance accelerometer operates on a technique where the capacitance ( $C$ ) of the internal sensing element ( $S$ ) changes in proportion to the applied acceleration ( $a$ ).

